Digital Filtering Technique in SB FDTD for SPP Propagation Modeling on Graphene Based Optical Structures

Md. Masud Rana[1], *Member, IEEE*, Md. Rabiul Islam[1], *Senior Member, IEEE*, Wei Xu[2], *Senior Member, IEEE*, and Jianguo Zhu[3], *Senior Member, IEEE*

[1]Rajshahi University of Engineering & Technology, Rajshahi-6204, Bangladesh, md.masud.rana@ruet.ac.bd
[2]State Key Laboratory of Advanced Electromagnetic Engineering and Technology, School of Electrical and Electronic Engineering, Huazhong University of Science and Technology, Wuhan, 430074, China, weixu@hust.edu.cn
[3]Faculty of Engineering & IT, University of Technology Sydney, Sydney, 2007, AU, Jianguo.Zhu@uts.edu.au

In this paper, a low pass digital filter in the surface boundary (SB) finite difference time domain (FDTD) is presented for the surface plasmon polariton (SPP) propagation modeling in the graphene based optical structures. The proposed digital filter in SB FDTD is able to eliminate the instabilities in time domain. The designed filter can be used as a boundary condition replacing perfectly matched layered, perodic boundary condition with the numerical methods for time domain analysis. The SPP propagation on different graphene structures in SB FDTD with the proposed filter is provided to validate it. The proposed digital filter in SB FDTD method can be applied in designing ultra-compact integrated devices for opto-electronics communication

Index Terms— FDTD, FIR, digital filter, graphene, SPP, propagation**.**

I. Introduction

RECENTLY research focus has concentrated on the optical coupling of surface plasmons between graphene sheets [1].The characteristics of surface plasmon polariton (SPP) modes in graphene sheets based optical devices such as plasmonic polarizers, optical splitter, nano-patch antenna, plasmonic interferometer and high-speed plasmonic modulators have been investigated theoretically and numerically [2]–[3]. However, for accurate designing of these graphene sheet based optical devices, efficient numerical technique is required. Till now, numerical methods such as finite element method (FEM), method of moment (MoM) and finite difference time domain (FDTD) method [4]–[5] have been proposed for the SPP propagation modeling of graphene sheet structures. Later, subcell FDTD and surface boundary condition (SBC) FDTD [6] have been developed for SPP propagation modeling in graphene structures in wide frequency band, but subcell FDTD cannot model infinite thin sheets accurately as it requires special type of perfectly matched layer (PML) [6]. In addition, SPP propagation in graphene sheet based optical structures is not presented with digital filtering technique in time domain. Though digital filtering techniques can be employed to improve the stability and performance of the graphene based optical structures. Digital filter techniques with FDTD methods in [7] has been applied to ensure stability and to eliminate the growth of parasitic solutions in the system, but did not applied in SB FDTD for the propagation modeling in the graphene structures.

In this paper, for the first time, digital filter in SB FDTD has been proposed for SPP propagation modeling in graphene based optical structures in time domain. It is found designed filters can replace existing boundary conditions such as perfectly matched layer (PML) with much success to eliminate the instabilities in time domain.

II. Digital Filtering Technique

To suppress the high frequency numerical error that affects the time-domain discrete field solution inside FDTD computational domain and to absorb the waves at edges altogether, finite impulse response (FIR) low pass filter (LPF) is used in the SB FDTD. The FIR LPF is based on the window technique. Let, the desired frequency response specification $H_d(\omega)$ and the corresponding unit sample response $h_d(\omega)$. Indeed, $h_d(\omega)$ is related to $H_d(\omega)$ by the Fourier transform relation,

$$X_d(\omega) = \sum_{n=0}^{\infty} x_d(n) e^{-j\omega n} \tag{1}$$

where,

$$x_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_d(\omega)\, e^{j\omega n} \tag{2}$$

Truncation of $x_d(n)$ to a length $M-1$ is equivalent to multiplying $x_d(n)$ by a "rectangular window ," defined as,

$$w(n) = \begin{cases} 1, & 0 \le n \le M-1 \\ 0, & otherwise \end{cases} \tag{3}$$

Multiplication of the window function $w(n)$ with $x_d(n)$ is equivalent to convolution of $X_d(\omega)$ with $H(\omega)$,where, is the frequency-domain representation (Fourier transform) of the window function, that is,

$$H(\omega) = \sum_{n=0}^{M-1} w(n) e^{-j\omega n} \tag{4}$$

Thus the convolution of $X_d(\omega)$ with $H(\omega)$ yields the frequency response of the (truncated) FIR filter. The Fourier transform of the rectangular window is,

$$H(\omega) = \sum_{n=0}^{M-1} e^{-j\omega n} = \frac{1-e^{-j\omega M}}{1-e^{-j\omega}}$$

$$= e^{-j\omega(M-1)/2}\,\frac{\sin(\omega M/2)}{\sin(\omega/2)} \tag{5}$$

This window function has a magnitude response and a piecewise linear phase. So, the filter's effect on a sequence (n) is described by the convolution theorem,

$$F\{\, x_d * h \,\} = F\{\, x_d \,\}\,.\; F\{\, h \,\} \tag{6}$$

$$Y(\omega) = X(\omega) \,.\, H(\omega) \tag{7}$$
and
$$y(n) = x_d(n) * h(n) \tag{8}$$
$$y(n) = F^{-1}\{X_d(\omega) \,.\, H(\omega)\} \tag{9}$$
where, the operators F and F^{-1} respectively denote the Fast Fourier Transform and the Inverse Fast Fourier Transform . Specifically, the convolution of $X_d(\omega)$ with $H(\omega)$ has the effect of smoothing $X_d(\omega)$.

III. FDTD Updating Equation with Low Pass Filter

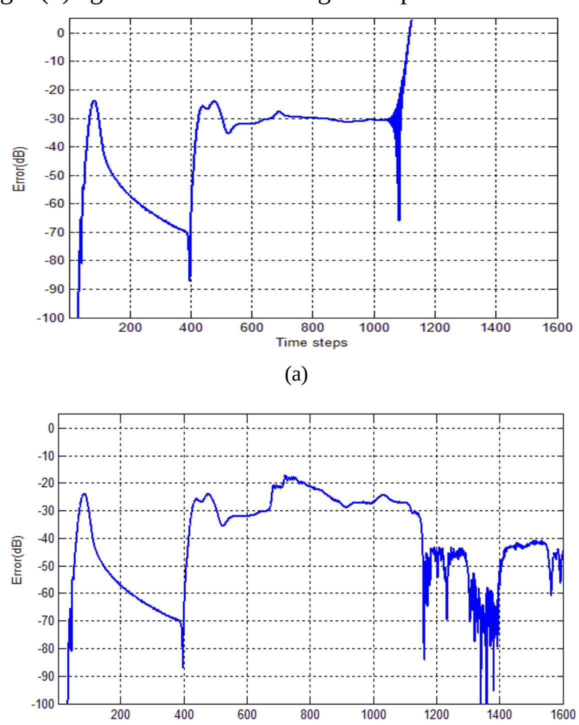
The proposed update filter is applied to the 1D problem space where the wave in the selected Yee grid cells are treated as a sequence $A(n)$ and then filtering is applied to the sequence. These steps are to be used in each time step after updating the fields for 1D problem space. For the E field at the boundary,

$$A_{ey_b}(n) = E_y^{n+1}(k_{i+\frac{1}{2}}) \tag{10}$$
$$B_{ey_b} = Filter(A_{ey_b}) \tag{11}$$
$$E_y^{n+1}\left(k_{i+\frac{1}{2}}\right) = A_{ey_b}(n) \tag{12}$$

Similar way, updating equation for the H field can be written.

IV. Numerical Results

At first, free space of 400×400 cells with a Gaussian pulse is considered and analyzed with the proposed method in time domain. The proposed filter is used as absorbing boundary condition (ABC) here. Fig. 1 (a) shows that the system is unstable after 1000 time steps. If a low pass filter is added with the system in conjunction with the pre-existing ABC and after 500 steps, then the system becomes stable as shown in Fig. 1(b) again while introducing some performance loss.



(a)

(b)

Fig. 1. (a) Unstable system, (b) Stable system with FIR-LPF

In this example, we simulate TM SPP wave on an infinite freestanding graphene by the means of the given Surface Boundary condition (SBC) method. The graphene sheet is positioned in the middle of the computational domain with 200 × 200 cells where the size of each cell is set to 0.15 × 0.15 . The time step is set to $\Delta = 3.56 \times 10^{-16}$ to meet the CFL stability condition. The computational space is terminated by a ten cells of PML in and by the proposed filter. A magnetic dipole is with a continuous sinusoidal waveform is for the excitation. The symmetry used in the excitation leads to a faster steady state response. The spatial distribution of at the time step 5000 when the fields reach steady state. Fig.2 clearly showing the SPP surface wave on the graphene layer. The SPP guided wavelength λ_{SPP} is easily extracted from the steady state electric field, which gives us $\lambda_{SPP} = 67 \times 0.15\ \mu m = 10.05\ \mu m$.

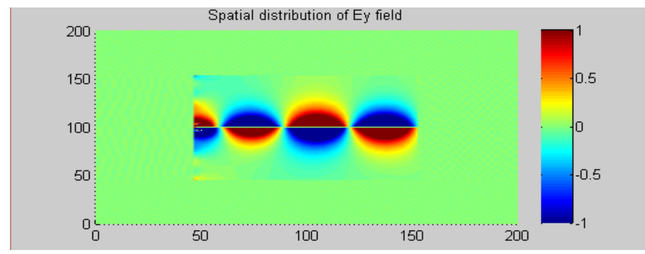


Fig. 2. SPP propagation with the proposed filter as absorbing boundary condition. Spatial distribution of at time step 2500 depicting TM SPP surface wave on the graphene layer with = 500 . The guided wave length is extracted from the field distribution with 47 layers of filter

V. Conclusion

FIR LPF digital filter technique with SB-FDTD has been proposed for the first time for the SPP propagation modeling in the graphene based optical structures. The digital filter in SB FDTD is able to eliminate the instability in time domain analysis. It is shown that designed filters can replace existing boundary conditions such as PML with much success.

References

[1] M. B. Hossain, M. Muktadhir, and M. M. Rana, "Multi-structural optical devices modeling using graphene," *Optik – Int. J. Light and Electro. Optics*, vol. 127, no. 15, pp. 5841-5851, Mar. 2016.

[2] H. Ju Li, L. L. Wang, Z. R. Huang, B. Sun, X. Zhai and X. Fei Li, "Simulations of multi-functional optical devices based on a sharp 90 bending graphene parallel pair," *J. Opt.*, vol. 16, 2014.

[3] W. B. Lu, W. Zhu, H. J. Xu, Z. H. Ni, Z. G. Dong and T. J. Cui, "Flexible transformation plasmonics using graphene," *Opt. Express*, vol. 21, no. 9, pp. 10475–10482, 2013.

[4] G. D. Bouzianas, N. V. Kantartzis, C. S. Antonopoulos, and T. D. Tsiboukis, "Optimal modeling of innite graphene sheets via a class of gener-alized FDTD schemes," *IEEE Trans. Magn*. vol. 48, pp. 379–382 2012.

[5] V. Nayyeri, M. Soleimani, and Omar M. Ramahi, "Modeling graphene in the finite-difference time - domain method using a surface boundary con-dition," *IEEE Trans. Antennas Propagat.*, vol. 61, no. 8, pp. 4176–4182, Aug. 2013.

[6] V. Nayyeri, M. Soleimani, and O. Ramahi, "Wideband modeling of graphene using the finite difference time-domain method," *IEEE Trans. Antennas Propagat*., vol. 61, no. 12, pp. 6107–6114, Dec. 2013.

[7] K. K. Mei, and J. Fang, "Superabsorption: a method to improve absorbing boundary conditions,". *IEEE Trans. Antennas Propag., vol.* 40, no. 9, pp. 1001-1010, 1992.